# Comment on "Quantum electrodynamics near a photonic band gap: photon bound states and dressed atoms"

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ABSTRACT

The applicability of the so-called isotropic and anisotropic complete photonic-band-gap (CPBG) models [S. John and J. Wang, Phys. Rev. Lett. **64**, 2418 (1990)] to capture essential features of the spontaneous emission (SE) of a fluorescent atom or molecule near a band-gap-edge of a CPBG structure is discussed. It is argued that, depending on the source position within a unit cell, the SE near the same CPBG edge can be either strongly enhanced or strongly depressed.

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In their influential article [1], John and Wang considered the quantum electrodynamics (QED) of an atom, minimally coupled to the radiation filed, in the presence of a complete photonic-band-gap (CPBG). In the later case, there is a frequency interval in which, independently of the photon direction and polarization, no photon modes can propagate. Such a QED vacuum differs significantly from the conventional QED vacuum. In the vicinity of a CPBG edge $\omega_c$ a number of exotic phenomena were predicted, among others, radical changes in the spontaneous emission (SE) and an anomalous Lamb shift. (It is worthwhile to notice that some of the exotic phenomena have been discussed much earlier in less known papers by Bykov [2].) Atom properties were shown to depend strongly on the exponent $\eta$ of the density of states (DOS) asymptotic

$$\rho(\omega) \approx \text{const}\, |\omega - \omega_c|^{\eta} \tag{1}$$

near the CPBG edge. Calculations involving photonic crystals in more than one dimension (1D) are notoriously difficult. Therefore, in order to determine $\eta$, John and Wang made use of approximations, subsequently employed in a number of recent discussions of the SE near a CPBG edge [3], and often called isotropic and anisotropic CPBG models. In the first model, the DOS near the band edge $\omega_c$ is obtained from an approximated dispersion relation of the CPBG material $\omega_{\mathbf{k}} \approx \omega_c + A(\mathbf{k} - \mathbf{k}_0)^2$, where $A \approx \omega_c/\mathbf{k}_0^2$ and $\mathbf{k}_0$ is a vector at the Brillouin zone boundary. The second model is then a slight generalization of the first one. Here we show that such approximations are rarely justified in a real photonic crystal (PC).

Real PC has a finite size and the presence of impurities is almost unavoidable. Both these features cause smoothing of the sharp features of the DOS near band edges. Even if the smoothing is neglected, already in the Wigner-Weisskopf approximation the SE of an isolated fluorescent atom (or molecule) in a fixed position $\mathbf{r}$ within the unit cell is determined by the local DOS (LDOS) and not the DOS [4]. Considerations based on the DOS can only be valid in the two hypothetical cases which are difficult to achieve: (i) when the atom is allowed to freely propagate within a CPBG structure, as in experiments with cavity QED, and (ii) when atoms are distributed homogeneously within the entire unit cell. Only then the averaged and not the local properties of the QED vacuum within the unit cell are probed. The fluorescent atoms are usually not distributed uniformly. If the atoms can be considered as independent and are only radiatively coupled to the crystal, the

measured SE is determined by the weighted average (with the atomic position probability distribution) of the LDOS. However, in contrast to the DOS, the LDOS asymptotic near a CPBG edge need not be described by Eq. (1) and if does, exponent $\eta$ depends on the position within a unit cell. For example, let us take a 1D structure of two alternating layers with their respective width ratio 3/2 and their respective dielectric constants 1 and 12. Let a unit cell be centered around the layer with the higher dielectric constant. In Table below the approximate values of $\eta$ are collected at the lower and upper edge of the first two gaps for 5 equidistant points within the unit cell, from the cell boundary ($A$) up to the cell center ($E$)[1].

| | A | B | C | D | E |
|---|---|---|---|---|---|
| $\eta_{1-}$ | 0.55 | $-0.35$ | $-0.39$ | $-0.49$ | $-0.5$ |
| $\eta_{1+}$ | $-0.44$ | $-0.44$ | $-0.44$ | $-0.49$ | 0.51 |
| $\eta_{2-}$ | 0.52 | $-0.38$ | $-0.42$ | $-0.51$ | 0.51 |
| $\eta_{2+}$ | $-0.47$ | $-0.46$ | $-0.44$ | $-0.32$ | $-0.51$ |

Depending on $\mathbf{r}$, the LDOS near the same CPBG edge can either diverge to infinity or decrease to zero [5]. Within a given frequency band, the LDOS as a function of $\mathbf{r}$ exhibits minima and maxima, their number depending on the order of the band starting from the lowest one. Behavior analogous to that of Eq. (1) can be expected only for $\mathbf{r}$ sufficiently away from the minima of the LDOS. Anyway, the value of $\eta$ is nonuniversal. The LDOS asymptotic is especially sensitive to $\mathbf{r}$ in the vicinity of the LDOS minima, where the LDOS approaches a constant limit and, unless at the exact position of a minimum, the dependence (1) breaks down. A shift in position by $10^{-4}$ of the unit cell length (i.e., a shift by one atom for optical photonic crystals) can cause a change in the LDOS by a factor of 3 or higher. In higher dimensions the issue is complicated further by the fact that the edge of a CPBG can be formed by different bands in different directions. To conclude, the LDOS asymptotic near a band edge can still have, under certain conditions, a form analogous to that described by Eq. (1), however, predictions derived from the hypothesis of a certain fixed value of $\eta$ (such as $\eta = 1/2$) [3] can have only limited application.

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[1]Fortran code to calculate the LDOS asymptotic at band edges is available upon request.

# References


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